Bi-Laplace done by 2-step Laplace

(the forth-and-back trick) -- 2D example

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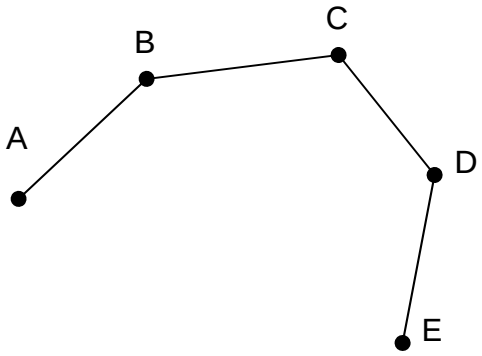

Bi-Laplace:

$$C' = (1-r_{Bi})*C + r_{Bi}\left((4/3)(\frac{B+D}{2}) + (-1/3)(\frac{A+E}{2})\right)$$

$$= (1-r_{Bi})*C + \left(\frac{2}{3}r_{Bi}\right)*(B+D) + \left(\frac{-1}{6}r_{Bi}\right)*(A+E) \quad (1)$$

Where r_{Bi} is the ratio multiplied to the Bi-Laplace Vector.

Two-Step Laplace:

$$C' = (1-r_2)\left[(1-r_1)*C + (r_1)*(\frac{B+D}{2})\right] + (r_2)\left[\frac{\left((1-r_1)*B + (r_1)*(\frac{A+C}{2})\right) + \left((1-r_1)*D + (r_1)*(\frac{C+E}{2})\right)}{2}\right]$$

Re-arrange:

$$C' = \left(1-r_2-r_1+(3/2)*(r_1r_2)\right)*C + \left(\frac{r_1+r_2-2(r_1r_2)}{2}\right)*(B+D) + \left(\frac{r_1r_2}{4}\right)*(A+E) \quad (2)$$

Compare (1) and (2) Hence,

$$(1-r_{Bi}) = \left(1-r_2-r_1+(3/2)*(r_1r_2)\right)$$

$$\left(\frac{2}{3}r_{Bi}\right) = \left(\frac{r_1+r_2-2(r_1r_2)}{2}\right)$$

$$\left(\frac{-1}{6}r_{Bi}\right) = \left(\frac{r_1r_2}{4}\right)$$

Solve→

$$\boxed{r_1 = -r_2}$$

$$r_2 = \sqrt{\frac{2}{3}r_3}$$

For example, when $r_3 = 0.1$

$r_1 = 0.2582$ $\quad r_2 = -0.2582$